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Modern Math: Logic, Probability, and Statistics

December 2024

The Art of Infinity: Fractals in African Culture

Fractals are geometric shapes containing detailed structure at arbitrary small scales; they are infinite complex patterns that are self-similar across a variety of scales. For example, fractals often arise in nature, appearing in snowflakes, coastlines, and mountain ranges due to their unique patterns. These intricate patterns can be described mathematically using fractal dimensions to analyze to what extent they are geometrically repeating. Fractals are also widely used in fields such as computer science and physics, offering in depth insight into their complex systems and patterns. By bridging the gap between human life and mathematics, fractals offer a profound way to explore the underlying structure and behavior of complex systems, marking them as a cornerstone of both scientific and artistic expansion.

This year, the Mathematical Association of America presented an article in their August/September edition of their Focus magazine regarding fractals in Africanist music. In 2012, choreographer Reggie Wilson, a native to New York City, was creating new material for a new dance set to perform in months to come; Wilson is known for his creation of modern dance while incorporating African traditions. As he was working through the choreographing process, he came to the realization that African features including rhythm, force, and tempo, did not easily map on the schema that normally categorizes common westernized dance. He used this ideology to hypothesize that “fractals were also prevalent in the performative cultures of Africa” (18). This spiked his curiosity to analyze fractals within Africanist music, addressing which

countries had the highest dimension, and overall, how it compared to westernized music. This exploration not only highlighted the mathematical complexity of Africanist traditions, but also underscored the unique cultural frameworks that distinguish them from western art. By combining choreography, music, and mathematics, Wilson's idea started to open new pathways for understanding the connection between culture and mathematics.

With this newfound understanding, Wilson brought in mathematical professionals to confirm his hypothesis: Claudio Gómez-González, professor of mathematics at Carleton College, Sidhanth Raman, scholar at University of California-Irvine, Siddharth Viswanath, student at Yale University, and Jesse Wolfson, professor of mathematics at University of California-Irvine. Together, this team combined their knowledge of mathematics and cultural studies to take a deeper dive into the fractal patterns of Africanist music. Their collaboration sought to provide a framework to analyze and quantify these patterns, once again bridging the gap between theoretical mathematics and African cultural expressions.

The team first addressed the steps to analyze the samples of music being used; they recognized that the Higuchi's method and the Hausdorff calculations would be necessary to quantify the data. First, Higuchi's method is a mathematical algorithm that computes fractal dimensions from time-series data like audio waves. It measures the length of a signal at different time scales and compares the measurements to determine the fractal dimension. Higuchi's method is particularly effective for identifying the intricate patterns and complexity within time-series data, such as audio signals. By calculating the fractal dimension, it provides a quantitative measure of how detailed a waveform appears at different scales, offering insight into its structural richness. This makes it a valuable tool for distinguishing between different types of signals and uncovering hidden patterns in musical compositions. With the information from

Higuchi's method, then the Hausdorff calculation is used to quantify the complexity or "roughness" of fractals. Fractals have non-integer dimensions, which reflect their intricate structure. The Hausdorff calculation complements Higuchi's method by providing a mathematical framework to assess the geometric properties of fractals. It helps in identifying the degree of irregularity and the scale at which these patterns exist within the music samples. Together, these methods enable the team to analyze the interplay between temporal and structural complexity, revealing unique characteristics of the audio data.

The researchers first tested on very simple rhythms like nursery rhymes. The simple waveforms did not produce fractal dimensions. The absence of fractal dimensions in simple rhythms aligned with expectations, confirming that the software could differentiate between basic and intricate patterns. This validated their software which got them ready to analyze the more complex african music. The experiment used 2 approaches: listening to the music samples and qualitatively identifying sections that seemed "fractal" based on rhythm and complexity, and then quantitatively calculating the fractal dimension using their software. This dual approach allowed the researchers to explore both the subjective and objective aspects of musical complexity. Listening to the samples provided an intuitive understanding of potential fractal sections, while the software offered precise numerical confirmation. Together, these methods created an outline for analyzing the intricate rhythms and patterns found in the African music samples. They then categorized the samples in different dimensions: least fractal being less than 1.02, average being between 1.03 and 1.07, and most being greater than 1.09. By assigning these thresholds, the researchers could systematically evaluate how the fractal dimensions correlated with the perceived intricacy and cultural context of the African rhythms.

Wilson provided 390 music samples across the African continent ranging from 28 different albums. For the results, South Africa lead the list with on average the highest fractal dimension in their music with 1.214. Also amongst the top countries were Senegal with an average of 1.189 and Congo with an average of 1.183. Some of the countries at the bottom of the list were Cameroon at an average of 1.1, Tanzania at an average of 1.076, and the lowest being Namibia at an average of 1.048. Most of the music samples Wilson predicted to be fractal had higher Hausdorff dimensions. These were songs with complex drumming and musical patterns. The results indicated a strong regional variation in musical complexity, with South Africa, Senegal, and Congo standing out for their intricate rhythms and layered compositions. In contrast, countries like Namibia and Tanzania exhibited simpler rhythmic patterns, corresponding to their lower fractal dimensions. Wilson emphasized that the prevalence of advanced drumming techniques and polyrhythms in the top-ranking countries likely played a significant role in their higher scores. This research not only highlighted the mathematical foundation of African music, but also provided a new perspective on how musical complexity can be quantified and appreciated across diverse cultural contexts.

The researchers expanded their study to modern day westernized music and tested 50 songs from Billboard top 100. While some songs did have high fractal dimensions, they did not nearly replicate the complex patterns the African traditional songs did. The researchers observed that many of the Western songs exhibited simpler, more repetitive structures compared to the intricate rhythms and polyrhythms found in African music. Again, despite the presence of some higher fractal dimensions in a few songs, the overall complexity was much lower than what was seen in the African samples. This highlighted a distinct difference in musical composition, with Western music often relying on more predictable and standardized patterns. The findings convey

that while modern Western music can be complex in its own right, it lacks the same level of intricacy that is present in African traditional music. This idea suggests that fractal structures in music may be influenced by specific cultural or traditional practices rather than being universal across all genres.

Overall, This research opens up new paths to be able to analyze songs through fractal geometry. It allows researchers to quantify the complexity and structure of music in ways traditional music theory might not be able to. It also opens the door to exploring how cultural and regional factors influence the complexity of music. Ultimately, this research not only enhances our understanding of music, but also bridges the gap between the arts and sciences, offering an insightful way to study the patterns that underlie musical creativity. Looking forward, this research provokes more conversation between culture and mathematics on how they intersect.

After reading the article and truly analyzing the strong complexities of the findings, I became motivated to keep investigating the connection between culture and mathematics. I sought out to explore the fractal dimension of African visual art to see if there was a connection between music and visual art dimensions by country; I also yearned to see if the fractal dimension of the music per country matched the visual art dimension. This idea intrigued me as it suggested a potential cultural thread connecting different artistic mediums through mathematical complexity; exploring the relationship between fractal dimensions in music and visual art could provide deeper insights into how cultural aesthetics manifest across various forms of expression. By comparing these dimensions by country, I yearned to uncover patterns that might reveal a shared underlying mathematical foundation within African artistic traditions.

To begin, I first had to understand how to measure fractality of visual art, as the process is somewhat different from audio fractality. To reiterate, fractal dimensions are calculated by

Hausdorff's dimension, which in short, is just a measure of fractality. I recognized the formula for calculating the dimensions to be as follows: $\text{Dimension} = \frac{\log(N)}{\log(S)}$, in which N represents the number of self-similar repeating patterns, and S represents the scale factor. For visual art, this meant examining patterns such as repeated geometric designs and scaling elements within the artwork. In-depth analysis of the art was necessary to identify and measure these self-similar features accurately. This approach allowed me to calculate fractal dimensions for visual art in a way that paralleled the method used for analyzing music. By applying the formula, I could quantify the complexity and repetition inherent in each piece, providing a new perspective on the mathematical structure of visual art. This method not only provided a numerical measure of complexity, but also offered insights into the artistic styles and cultural significance embedded within the patterns. By comparing these dimensions across different regions, I aimed to explore whether visual art shares similar fractal characteristics with the music of the same cultural origins: this connection could reveal a deeper, cross-disciplinary understanding of how fractal geometry influences artistic expression.

After fully recognizing the steps to take to analyze the images, I then found the most famous artwork across several African countries to find their fractal dimensions. I first started in South Africa with Ndebele Wall Paintings. Ndebele wall paintings are geometric murals known for their vibrant and iconic form of art created by the Ndebele people of South Africa. Traditionally, these paintings are done by women and adorn the exterior walls of homes, serving as a form of expression and identity. The art is characterized by intricate geometric patterns, bold colors, and striking symmetry, often conveying messages of cultural pride, social status, or familial heritage. The practice dates back to the 18th century and has evolved over time, incorporating modern influences while preserving its cultural significance. Beyond their

aesthetic appeal, these paintings play a crucial role in community rituals and celebrations, symbolizing resilience and unity within the Ndebele culture. These fractal-like designs adorn homes and reflect cultural identity, family heritage, and rites of passage. The fractal symmetry and self-similarity in the patterns are clear examples of the recursive geometric shapes found in fractal mathematics. I found one of the most famous designs that to date sells as a mural to homes across the country. After analyzing the scale and number of self-similar repeated patterns, I calculated the image to have a fractal dimension of approximately 1.4. This is a high dimension similar to the South African music dimension: this data intrigued me to keep searching. I then moved a bit more north to the Democratic Republic of Congo to analyze patterns called Kasai Velvet. Kasai Velvet, also known as Shoowa textiles, are intricately designed textiles created by the Kuba people of the Democratic Republic of Congo. These textiles are traditionally handwoven from raffia palm fibers and embroidered with complex geometric patterns. Historically, Kasai Velvet has been used for ceremonial purposes, such as dowries, funerals, and rites of passage, symbolizing wealth, status, and cultural identity. The designs often incorporate improvisation, reflecting the creativity and individuality of the artists. These intricate designs often include interlocking diamonds, triangles, and squares that mimic fractal structures. Like before, I chose a select famous pattern that is commonly auctioned off. Its calculated fractal dimension came out to roughly 1.25, also resembling the fairly high music dimension the Democratic Republic of Congo had. Thirdly, I moved up to the Saharan area to study the Mud Cloths of Mali. Mud cloths, or otherwise known as bogolanfini, are a traditional textile art form originating from Mali, crafted primarily by the Bambara people. These fabrics are made using a labor-intensive process where cotton cloth is dyed with fermented mud to create intricate, symbolic patterns. The use of lines, dots, and symbols arranged in self-similar patterns across the

cloth is a hallmark of this traditional African art form. Historically, mud cloths have been used in ceremonies, including weddings and initiations, and as a means of storytelling, with each pattern and design carrying specific meanings. They often serve as a way to communicate social status, identity, or spiritual beliefs within the community. Today, mud cloths remain a cherished art form, blending traditional practices with contemporary designs, and symbolize the enduring cultural heritage of Mali. After finding another particularly famous pattern and examining it for its fractal dimension, I calculated it has a fractal dimension of approximately 1.3, which in comparison to the other examples, has a high fractal dimension similarly to its music dimension. I then analyzed West Africa by looking at the Tikkar designs of Cameroon. The Tikkar people create designs that feature repeating, fractal-like patterns; the designs often include spirals and circular forms, which are key examples of self-similarity in African fractal art. These designs often feature intricate geometric patterns and symbolic motifs that are prominently used in textiles, pottery, masks, and sculptures. Historically, Tikar art played a vital role in ceremonial and ritualistic contexts, serving as a means to honor ancestors, convey social status, and express beliefs. The patterns are rich in symbolism, with many representing spiritual ideas, natural elements, or communal narratives. Today, Tikar designs are celebrated as an integral part of Cameroon's cultural heritage, blending traditional artistry with contemporary influences while preserving their historical significance. As I did for the previous countries, I found a pattern that is quite prominent to the Tikkar culture and surveyed it for its scale and self-similar repeating patterns; I calculated its fractal dimension to be 1.15, which is almost identical to its respective music dimension. Lastly, I went back south to analyze Namibia and its known rock paintings. Namibia's rock paintings, particularly those at Twyfelfontein, are among the oldest and most significant examples of prehistoric art in Africa. Created by the San people thousands of years

ago, these paintings primarily depict animals, human figures, and abstract symbols, providing insight into the daily life, spiritual beliefs, and hunting practices of the time. The paintings are believed to have had a ritualistic purpose related to the San people's spiritual connections with the natural world and their belief in animal spirits. Twyfelfontein, a famous world heritage site, contains over 2,500 rock engravings, making it one of the largest concentrations of rock art in Southern Africa. These artworks not only serve as a visual record of the past, but also symbolize the rich cultural heritage and deep spiritual traditions of Namibia's indigenous people. When analyzing one of the most famous rock paintings on the site, consisting of various animals, I calculated the fractal dimension to be roughly 1.02, signifying a low fractal dimension similarly to its respective music dimension.

After testing the most famous art forms and patterns across different cultures and geographical regions of the continent of Africa, I was able to note that the fractal dimensions of the visual art, calculated by Hausdorff's dimension, was relative to each country's respective music fractal dimension calculation. This finding suggests that the intricate patterns and complexity in both visual art and music are culturally linked, with similar fractal dimensions indicating a shared component across different forms of expression. The correlation between the fractal dimensions of visual art and music emphasizes the role of mathematics in cultural creativity, reflecting how cultural traditions may inherently favor complexity and self-similarity more than others. In countries like South Africa and the Democratic Republic of Congo, both the visual art and music exhibited higher fractal dimensions, pointing to a common appreciation for layered, intricate patterns. Conversely, regions with simpler artistic expressions, such as Namibia, also showed lower fractal dimensions in both their visual art and music. This parallel

analysis offered a compelling case for the presence of fractal geometry as a fundamental element in African artistic expression across multiple mediums.

In conclusion, between Wilson's original hypothesis regarding fractals in African music, and my continued study analyzing fractals in African art, there seems to be a true connection between mathematics and African culture. This connection highlights how fractal geometry, a mathematical concept, not only appears in music, but also in the visual art forms of various African cultures. The similarities in fractal dimensions across different countries suggest that African art, whether through music or visual design, often embodies patterns of complexity and self-similarity that resonate with the principles of fractal geometry. Ultimately, this research underscores the universality of mathematical concepts in cultural expression, bridging art, mathematics, and tradition.

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